

Testaufgaben (Lösungen)

1) a) $\underline{\underline{-a^{12}}}$ b) $\underline{\underline{b^{12}}}$ c) $\underline{\underline{-c^{12}}}$ 2) $A = \underline{\underline{9}}$

3) a) $\underline{\underline{r^{k+1}}}$ b) $\underline{\underline{r^{k-1} - r = r(r^{k-2} - 1)}}$ c) $\underline{\underline{-4a^2 + 9b^2 - 6bc + c^2}}$

4) a) $\underline{\underline{\frac{2}{3}u}}$ b) $\underline{\underline{\frac{b^5 c^2}{a^4}}}$ c) $b \sqrt[10]{3^8 b^3} = b \sqrt[5]{3^4} \cdot \sqrt[10]{b^3}$ d) $\underline{\underline{\sqrt{|a-b|}}}$ e) $\underline{\underline{2 - 2\sqrt{1-x^4}}}$

f) $z^{\frac{1}{3} + \frac{1}{6} - \frac{4}{6} - \frac{2}{18}} = z^{-\frac{5}{18}} = \frac{1}{\sqrt[18]{z^5}}$ g) $-u$ h) $2st + \frac{s^2t - 2st^2}{t-s} = \frac{2st^2 - 2s^2t + s^2t - 2st^2}{t-s} = \underline{\underline{\frac{-s^2t}{t-s}}}$

i) $\frac{y^2 - x^2}{x^2 + y^2 - 2xy} = \frac{(y-x)(y+x)}{(y-x)^2} = \underline{\underline{\frac{y+x}{y-x}}}$

5) a) $\underline{\underline{2 \lg |p| - \lg 2 - 4 \lg |r|}}$ b) $\underline{\underline{\frac{1}{4} \ln x + \frac{1}{12} \ln a - \frac{2}{3} \ln |c|}}$

6) a) $\ln \frac{e^6 2^3 e^3}{8 e^{10}} = \ln e^{-1} = \underline{\underline{-1}}$ b) $\lg \frac{u^2 2^3}{4^3 v^3 5^3 u^3} = \lg \frac{1}{2^3 v^3 5^3 u} = \lg \frac{1}{2^3 v^3 5^3 u} = \lg \frac{1}{10^3 v^3 u} = \underline{\underline{-\lg(v^3 u) - 3}}$

7) $(a^4 - 2a^3 + a^2 + 6a - 12) : (a^3 - 3) = \underline{\underline{a - 2 + \frac{a^2 + 9a - 18}{a^3 - 3}}}$

$$\begin{array}{r} -(a^4 - 3a) \\ - 2a^3 + a^2 + 9a - 12 \\ - (-2a^3 + 6) \\ a^2 + 9a - 18 \end{array}$$

8) a) $x^2 - 6x + 9 - x - 20 - x - 37 = 0$

$$x^2 - 8x - 48 = 0 \rightarrow x_1 = 12 \quad x_2 = 4$$

b) $\frac{24 - 3x - (4x - 22) - (x - 2)}{6} = 0$

$$24 - 3x - 4x + 22 - x + 2 = 0$$

$$-8x = -48 \rightarrow x = 6$$

c)

$$\begin{array}{l} 2a - 3a + 12 \cdot 5 - 12a = 24 - 18a - 18 \cdot 7 \\ 5a = -60 + 24 - 126 = -162 \\ a = \frac{-162}{5} = \underline{\underline{-32,4}} \end{array}$$

d)

$$\frac{x^2 + 2}{(x-2)(x+2)} - \frac{x+1}{4(x-2)} - \frac{5x-3}{8(x+2)} = 0 \cdot 8(x-2)(x+2)$$

$$8x^2 + 16 - 2(x^2 + 3x + 2) - (5x^2 - 10x - 3x + 6) = 0$$

$$x^2 + 7x + 6 = 0 \Rightarrow x_{\frac{1}{2}} = -\frac{7}{2} \pm \sqrt{\frac{49}{4} - \frac{48}{4}}$$

$$\underline{\underline{x_1 = -1 \quad x_2 = -6}}$$

e)

$$x + 9 + x + 2 - 2\sqrt{(x+2)(x+9)} = 4x - 27$$

$$-2\sqrt{x^2 + 11x + 18} = 2x - 38$$

$$\sqrt{x^2 + 11x + 18} = -x + 19 \quad ()^2$$

$$x^2 + 11x + 18 = x^2 + 361 - 38x$$

$$49x = 343$$

$$x = \underline{\underline{7}}$$

Probe:

$$\sqrt{16} - \sqrt{9} = \sqrt{1}$$

$$4 - 3 = 1$$

$$f) \quad \begin{aligned} \lg(u^2 + 5u + 6) &= \lg(10u) \\ u^2 + 5u + 6 &= 10u \\ u^2 - 5u + 6 &= 0 \end{aligned}$$

$$u_{1/2} = \frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{24}{4}} \quad \underline{u_1 = 3} \quad \underline{u_2 = 2}$$

$$\text{Probe: } u = 3: \quad \begin{aligned} \lg 5 + \lg 6 &= 1 + \lg 3 \\ \underbrace{\lg 5 + \lg 2}_{=1} + \lg 3 &= 1 + \lg 3 \end{aligned}$$

$$u = 2: \quad \begin{aligned} \lg 4 + \lg 5 &= 1 + \lg 2 \\ \lg 2 + \underbrace{\lg 2 + \lg 5}_{=1} &= 1 + \lg 2 \end{aligned}$$

$$9) a) \quad \begin{aligned} e^{-\lambda t} &= 1-y & | \ln(\) \\ -\lambda t &= \ln(1-y) \\ t &= \underline{\underline{-\frac{1}{\lambda} \ln(1-y)}} \end{aligned}$$

$$b) \quad \begin{aligned} -\left(\frac{x}{b}\right)^3 &= \ln \frac{f}{A_o} & | \cdot (-1) \\ \left(\frac{x}{b}\right)^3 &= \ln \frac{A_o}{f} \\ \frac{x}{b} &= \sqrt[3]{\ln \frac{A_o}{f}} \\ x &= \underline{\underline{b \sqrt[3]{\ln \left(\frac{A_o}{f}\right)}}} \\ b &= \underline{\underline{\frac{x}{\sqrt[3]{\ln \left(\frac{A_o}{f}\right)}}}} \end{aligned}$$

$$c) \quad \begin{aligned} R = s \cdot \frac{v-1}{v^n - 1} & \quad \frac{s}{R}(v-1) + 1 = v^n & | \ln(\) \\ \ln\left(\frac{s}{R}(v-1) + 1\right) &= n \cdot \ln v \\ n &= \underline{\underline{\frac{\ln\left(\frac{s}{R}(v-1) + 1\right)}{\ln v}}} \end{aligned}$$

$$d) \quad \begin{aligned} a^2 - 2b \cos \gamma \cdot a + b^2 - c^2 &= 0 \\ a_{1/2} &= \underline{\underline{b \cos \gamma \pm \sqrt{b^2 \cos^2 \gamma - b^2 + c^2}}} \\ \cos \gamma &= \frac{a^2 + b^2 - c^2}{2ab} \\ \gamma &= \arccos \frac{a^2 + b^2 - c^2}{2ab} \end{aligned}$$

$$e) \quad \begin{aligned} \left(\frac{1}{1+i}\right)^n &= \frac{K_o}{K_n} \Rightarrow (1+i)^n = \frac{K_n}{K_o} \\ n \cdot \ln \frac{1}{1+i} &= \ln \frac{K_o}{K_n} & 1+i = \sqrt[n]{\frac{K_n}{K_o}} \\ n &= \underline{\underline{\frac{\ln \frac{K_n}{K_o}}{\ln(1+i)}}} & i = \underline{\underline{\sqrt[n]{\frac{K_n}{K_o}} - 1}} \end{aligned}$$

$$f) \quad \begin{aligned} d &= \sqrt{\frac{(x-x_o)^2}{a^2} + y^2} & d^2 - y^2 &= \frac{(x-x_o)^2}{a^2} \\ y^2 &= d^2 - \frac{(x-x_o)^2}{a^2} & a^2 &= \frac{(x-x_o)^2}{d^2 - y^2} \\ y &= \pm \sqrt{d^2 - \frac{(x-x_o)^2}{a^2}} & a &= \pm \sqrt{\frac{(x-x_o)^2}{d^2 - y^2}} \\ x - x_o &= \pm \sqrt{a^2(d^2 - y^2)} \Rightarrow \underline{\underline{x_o = x \mp \sqrt{a^2(d^2 - y^2)}}} \end{aligned}$$

$$10a) \quad \begin{aligned} 3x - 4y &= 12 \\ 2x + 2y &= 22 & | \cdot 2 \\ \hline 7x &= 56 \\ \Rightarrow x &= 8 \\ 24 - 4y &= 12 \Rightarrow y = 3 \\ \underline{\underline{x = 8; y = 3}} \end{aligned}$$

$$b) \quad \begin{aligned} xy^2 - 2y + 6 &= 0 & x(y-1) &= 0 \Rightarrow x=0 \quad \vee \quad y=1 \\ x=0 &\Rightarrow -2y + 6 = 0 & & \\ y &= 3 & \underline{\underline{x_1 = 0; y_1 = 3}} & \\ y=1 &\Rightarrow x - 2 + 6 = 0 & & \\ x &= -4 & \underline{\underline{x_2 = -4; y_2 = 1}} & \end{aligned}$$

$$\begin{array}{rcl}
10 \text{ c)} & 7x + 42y - (27x - 144) & = 6y - 2x \\
& -18x + 36y & = -144 \\
& 5x + 2 - (8x + 14y + 12) & = 18y - 9x \\
& 6x - 32y & = 10 \\
\Rightarrow \text{I:} & x - 2y & = 8 \\
\Rightarrow \text{II:} & 3x - 16y & = 5 \\
-3 \text{ I} + \text{II:} & -10y & = -19 \\
\Rightarrow & x - 3,8 & = 8
\end{array}
\quad
\begin{array}{l}
y = +1,9 \\
x = 11,8
\end{array}$$

11 a) $y = 2x + 3$ b) $y = -x + 5$ c) $y = -\frac{3}{4}x + \frac{7}{2}$

12) Skizzen sind ohne Grafikrechner anzufertigen und können mittels Grafikrechner selbst überprüft werden.

13a) x liegt im III. oder IV. Quadranten b) x liegt im II. oder III. Quadranten c) x liegt im I. oder III. Quadranten

14a) $2 \cos \alpha$ b) $\cos^2 \alpha$ c) $\frac{1}{\sin \beta}$ d) $\tan^2 \beta$

e) $\begin{cases} 0,5 & \text{für } a > 0 \\ -0,5 & \text{für } a < 0 \end{cases}$ f) 0,9771 g) -0,2929 h) 0,1668

15a) $c = 5 \text{ m}$ b) $c = \sqrt{a^2 + b^2 - 2ab \cos \gamma} \approx 19,59 \text{ m}$ c) $c = \frac{b \cdot \sin \gamma}{\sin \beta} \approx 18,30 \text{ m}$
 $a = \frac{b \cdot \sin (180^\circ - \gamma - \beta)}{\sin \beta} \approx 27,19 \text{ m}$

16) $\vec{u} = -\vec{j} + 7\vec{k}$; $\vec{v} = 7\vec{i} - 10\vec{j}$ $\vec{a} \cdot \vec{b} = -16$

17 a) unbest. div. b) $b_n \rightarrow 0$ c) unbest. div. d) $\text{div. } d_n \rightarrow -\infty$ e) $e_n \rightarrow 0$ f) $b_n \rightarrow -4$

18 a) ∞ b) $\frac{3}{2}$ c) $-\frac{1}{2}$ f) 0

19a) $y' = 10x^4 - 12x$ b) $y' = \frac{1}{\sqrt{2x-1}}$ c) $y' = (1-x)e^{-x}$

20a) $e^x - 3x^4 + c$ b) $\ln 4 - 1$ c) $\frac{1}{6}$